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Elastic deformation of the rotating functionally graded annular disk with rigid casing

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Abstract An accurate solution for a rotating functionally graded annular disk is presented. Material properties of the present annular disk are assumed to be graded in the radial direction according to a simple exponential-law distribution. The inner surface of the disk is pure metal whereas the outer surface of the disk is pure ceramic. The boundary condition of rigid casing is considered herein, that is the vanishing of the radial displacement at the outer surface. The boundary condition at the inner surface of the disk is taken to be vanishing either radial displacement or radial stress. Analytical solutions for the elastic deformation of the rotating functionally graded annular disks subjected to these boundary conditions are obtained. Numerical results for radial displacement, circumferential and radial stresses are presented. Comparisons between the different rotating homogeneous and functionally graded annular disks are made at the same angular velocity. The results show that distributions of stresses and displacement through the radial direction of the rotating annular disk vary with different parameters.

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Introduction

The analysis of rotating annular disks is an important subject for many applications in mechanical engineering. Among these high-speed gears, turbine rotors, flywheels, disc brakes, disk drives, circular saw blades and shrink fits can be mentioned. The research on them is always an important topic and their benefits have been included in many references [1, 2]. The problem of determination of stresses and displacement in rotating annular disks subjected to different boundary conditions is important for an efficient design and material usage in many industrial applications. Most of the research works are concentrated on the analytical solutions of rotating disks with simple cross-section geometries of constant thickness and specifically variable thickness. The material density of these rotating disks is taken to be either constant or specifically variable. The analytical elasticity solutions of such rotating disks can be mainly found in the literature.

Gamer [3] has studied the analytical solution of elastic-perfectly plastic rotating disks of constant thickness and density by using Tresca's yield condition. Gamer [4, 5] has also studied the analytical solutions of such disks with a linear strain-hardening material behaviour using the same yield condition. Güven [6] has extended this work to rotating disks of variable thickness and density and obtained their analytical solution. You and Zhang [7] have obtained the approximate analytical solution for a rotating solid disk of uniform thickness. You et al. [8, 9] have numerically studied rotating solid disks of uniform thickness and constant density and annular disks of variable thickness and variable density. Eraslan [10, 11], and Eraslan and Orcan [12] have analytically studied rotating disks of exponentially varying thickness and of linearly strain hardening material. In a recent paper, Zenkour and