# ESHELBY'S TECHNIQUE FOR MODELING INTERACTION MECHANICS IN ADAPTIVE STRUCTURES

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## ABSTRACT

The purpose of this paper is to summarize previous work on modeling the interactions between embedded microdevices and the host in adaptive structures using Eshelby's classical equivalent inclusion methods. Some of the existing methods of analysis are discussed briefly. Eshelby's technique [Eshelby, 1957] offers a unified method to address both the sensors and the actuators. Sensors are treated as actuators without real induced actuation strains. The advantages of this method are shown through an example of a "smart" structure consisting of a simply supported beam with embedded arrays of microdevices. Preliminary analytical results for piezoelectric devices embedded in an isotropic host provide important clues regarding the change in the vibrational characteristics of the structure. Eshelby's methods also help to estimate the strain concentration factor in the host due to the presence of the embedded devices, and provide important inputs for designing a reliable adaptive structure.

## 1. INTRODUCTION

Adaptive structures and "smart" materials have been of great interest to researchers and engineers in science and engineering disciplines for the past decade. An adaptive structure is a structure with distributions of actuators, sensors, and data processing capability to respond to a sensed stimulus. The behavior of the structure is modified, tuned, and controlled via the distributed sensors and actuators, which are termed the active elements, for the purpose of this discussion. Sensors or actuators are either bonded to the structure or embedded within the structure. The possible applications of adaptive structures include space stations, aircraft structures, rotorcraft, satellites, robots, automobiles, civil and marine structures, and bioengineering structures.

There have been numerous studies in the literature for modeling the interactions between devices and hosts in smart structures. For example, the response of adaptive structures with laminated assemblies of piezo-electric wafers/films in composite beams have been analyzed by pin force models [Crawley and de Luis, 1987; Lin and Rogers, 1992], simple beam models [Bailey and Hubbard, 1987; Crawley and Lazarus, 1991], large deformation beam theory [Im and Atluri, 1989], laminate analysis [Crawley and Lazarus, 1991], and small-deformation as well as large-deformation one-dimensional eigen-function approximations [Lin and Rogers, 1992;Crawley and de Luis; 1987].

Embedded cylindrical devices such as fiber optic sensors have been analyzed using displacement function methods [Sirkis, 1993; Dasgupta and Sirkis, 1992; Carman and Reifsnider, 1992; Pak, 1992]. Variational methods have also been developed for solving the coupled boundary value problems in adaptive structures. These include the Rayleigh Ritz method [Hagood et al, 1990], finite element methods [Allik and Hughes, 197 ,Tzou and Tseng, 1990; Ha et al, 1991; Robbins and Reddy, 1991], and a strain energy method [Wang and Rogers, 1991]. Most of these models address situations where the size of the device is of the same length scale as the surrounding host structure. The focus of the present paper is on microscale devices which are small compared to the characteristic dimensions of the host.

In this paper we summarize recent results from previous work on modeling the interaction mechanics between embedded microdevices and the host structure using Eshelby's "equivalent inclusion method" for modeling the perturbation of a uniform applied far field strain, by an ellipsoidal inhomogeneity. Also, the equivalent inclusion method provides enough information about the possibility of nucleating damage in the device, or in the host, or at the interface, due to stress concentrations under external or internal loads. This method is illustrated here for the dynamic behavior of a simply supported beam made of ALPLEX plastic containing two rows of devices. The system equation of motion is solved and the Rayleigh quotient is used to study the change in the natural frequency of the structure due to harmonic excitation of the actuators.

## 2. PROBLEM STATEMENT

As shown schematically in Figure 1, two rows of uniformly spaced micro-devices are assumed to be embedded in the beam at a distance d/2 symmetrically about the neutral plane of the beam. In order to investigate the influence of the devices on the vibrational characteristics of the beam, the change in the natural frequency of the beam is studied. As the beam flexes, every alternate device in each row acts as a sensor and the outputs are used in a closed-loop feedback circuit to actuate the active half of the opposite row of devices. The actuation strain is assumed to be opposed in sign to the bending strain for all actuation. The result is an apparent stiffening of the beam and an accompanying increase in the natural frequency $\omega$, if all losses in the system are ignored. The aim in this study is to generate the electro-mechanical interaction information, necessary for combining the device response with that of the host beam, in an integrated dynamical equation of the adaptive structure.

Several simplifying assumptions are made in this approximate analytical study. Euler-Bernoulli beam theory is assumed to apply. Each embedded micro-device is assumed to be a piezoelectric micro-cylinder of elliptical cross-section, whose polarization axis is oriented along the length of the beam. The length scale of each device is limited to at least an order of magnitude less than the beam. Hence, the bending strain is assumed to be approximately uniform over the length scale of the device. This approximation greatly simplifies the eigenstrain solution. Further, each device is assumed to be embedded far enough below the free surface of the beam such that Eshelby's eigenstrain solution for infinite domains is applicable. Finally, the distance between neighboring devices is assumed to be large enough to prevent mutual interactions. Thus, this solution is only valid for dilute distributions of micro-devices.

As a result of the assumptions presented above, each micro-device is approximated to act like elastic heterogeneities embedded in an infinite-dimensional host structure. Perfect bonding is assumed at the interfaces. The sensor/actuator material is assumed to be PZT-5H. All materials are approximated to be linear and mechanically isotropic. The linearizing assumption limits the validity of this approximate analysis to small excitation voltages and small deformations. The assumption of mechanical isotropy is an acceptable approximation for most PZT materials.

## 3. ANALYSIS

Eshelby's classical equivalent-inclusion technique is applied to obtain the elastic interaction fields, both in the device and in the host, under external applied loads and under internal actuation loads. External loads are handled through Eshelby's fictitious eigenstrain technique. Internal actuation loads are treated as a real eigenstrains and are obtained from the linearized, isothermal, coupled electro-mechanical constitutive models given below. The difference between PZT sensors and actuators in the present analytical context is that the sensor only has a fictitious eigenstrain due to external loads, while the actuator has both fictitious and real eigenstrains.

The linearized, isothermal, coupled electro-mechanical constitutive model is [Ikeda, 1990]:

$$\vec{\sigma} = \underline{C}\,\vec{\epsilon} - \underline{h}^T \vec{E} \qquad (1)$$
$$\vec{D} = \underline{h}\,\vec{\epsilon} + \underline{e}\,\vec{E}$$

where $\vec{\epsilon}$ is the total strain vector including mechanical as well as electro-mechanical contributions, $\vec{\sigma}$ is the mechanical stress vector, $\vec{E}$ is the electrical field vector, $\vec{D}$ is the electrical displacement vector. $\underline{C}$ is the mechanical stiffness tensor, $\underline{h}$ is the piezoelectric coupling tensor indicating the stress caused by completely constrained excitation of the PZT material under a unit applied electrical field, $\underline{e}$ is the dielectric tensor for the PZT material. Arrows over a quantity are used to denote vector quantities, while an underscore is used to denote tensor quantities. Clearly, only the mechanical portion of this constitutive model applies to the ALPLEX host material.

Eshelby's method is based upon postulating an equivalent inclusion with a fictitious eigenstrain which has the same stress field as the real heterogeneity, under both external loads and internal actuation strains. Thus, in the heterogeneity:

$$\vec{\sigma}^0 + \vec{\sigma}' = \underline{C}^D\,(\vec{\epsilon}^0 + \vec{\epsilon}' - \vec{\epsilon}^r)$$
$$= \underline{C}^H\,(\vec{\epsilon}^0 + \vec{\epsilon}' - \vec{\epsilon}^*) \qquad (2)$$

where $\vec{\epsilon}^* = \vec{\epsilon}^r + \vec{\epsilon}^f$ ; superscripts D and H on the stiffness indicate the PZT device and the ALPLEX host, respectively; superscripts 0, ′ , r, f and * on the stress and strain terms indicate applied far-field value, perturbation due to the presence of the heterogeneity, real actuation eigenstrains, fictitious eigenstrains due to external loading, and total eigenstrains, respectively.

The real actuation eigenstrain is obtained from Equation (1) as:

$$\vec{\epsilon}^r = \underline{d}^T \vec{E} \qquad (3)$$
$$\text{where} \quad \underline{d} = \underline{h}\,\underline{S}^D$$

$\underline{d}$ represents the free-expansion of the piezoelectric actuator for a unit applied electric field and $\underline{S}^D$ is the compliance tensor of the device material.

The total eigenstrain is now related to the disturbance strain by Eshelby's strain concentration tensor $\underline{S}^E$:

$$\vec{\epsilon}' = \underline{S}^E\,\vec{\epsilon}^* = \underline{S}^E\,(\vec{\epsilon}^r + \vec{\epsilon}^f) \qquad (4)$$

Explicit forms for Eshelby's tensor are readily available in the literature for embedded isotropic heterogeneities of ellipsoidal geometries.

Substituting Equation (4) in Equation (2), we obtain:

$$\underline{C}^D\,(\vec{\epsilon}^0 + \underline{S}^E\vec{\epsilon}^* - \vec{\epsilon}^r) = \underline{C}^H\,(\vec{\epsilon}^0 + \underline{S}^E\vec{\epsilon}^* - \vec{\epsilon}^f - \vec{\epsilon}^r) \qquad (5)$$

Equation (5) can now be solved for the unknown fictitious eigenstrain $\vec{\epsilon}^f$ in terms of the applied external bending strain $\vec{\epsilon}^o$ and the real actuation eigenstrain $\vec{\epsilon}^r$ . If the applied strain is uniform, so is the fictitious eigenstrain.

Now, the mechanical and electrical energy terms are computed to obtain the stiffening of the structure under harmonic excitation, through a suitable variational scheme. The variational principle is a generalized form of Hamilton's principle, and may be written as [Tiersten, 1967];

$$\delta \left[ \int_{t_0}^{t_1} (L + W)\, dt \right] = 0 \qquad (6)$$

where the Lagrangian function L is the difference between the kinetic energy T and the electric enthalpy H. The work term, W, includes the potential of all applied mechanical loads and the electrical charges. Thus, using the definition of electric enthalpy [Tiersten, 1967]

$$\begin{aligned} L+W = -\int_V \Big( &\left(\tfrac{1}{2}\right) \vec{\epsilon}^T \underline{C}\, \vec{\epsilon} + \left(\tfrac{1}{2}\right) \vec{E}^T \underline{e}\, \vec{E} + \vec{\epsilon}^T \underline{h}\, \vec{E} + \\ &\left(\tfrac{1}{2}\right) \rho \omega^2 \vec{u}^T \vec{u} \Big)\, dV - \int_A \phi\, \vec{n}^T ( \underline{h}\, \vec{\epsilon} + \underline{e}\, \vec{E} )\, dA \end{aligned} \qquad (7)$$

where $\omega$ is the natural frequency, $\rho$ is the density of host or device, $\vec{u}$ is the displacement field, $\phi$ is the resulting potential, A is the surface area, and $\vec{n}$ is the unit outward normal vector.

The variation of (L+W) yields stationary solutions for which, the Rayleigh quotient can be presented as [EerNisse, 1967]:

$$\omega^2 = \frac{\int_V (\vec{\epsilon}^T \underline{C}\, \vec{\epsilon})\, dV + \int_\Omega (\vec{E}^T \underline{e}\, \vec{E})\, dV}{\int_V \rho\, \vec{u}^T \vec{u}\, dV} \qquad (8)$$

This equation has been used to predict the natural frequencies of smart structures with embedded actuators [Alghamdi and Dasgupta, 1993]. The principal results are summarized below.

The first integral in the numerator is the mechanical energy, and is given as [Dasgupta and Alghamdi, 1992]

$$U_{mech} = \frac{1}{2} \int_V \vec{\epsilon}^{0^T} \underline{C}^H \vec{\epsilon}^0\, dV + \frac{1}{2} \int_\Omega \vec{\epsilon}^{*^T} \underline{C}^H \underline{S}^E \vec{\epsilon}^*\, dV \qquad (9)$$

where $\vec{\epsilon}^*$ is obtained from Equation (5).

In order to perform the integrations in Equations (8) and (9), all that remains now is to assume explicit representations for the applied flexural strain field, and the actuation eigenstrain. For example, in the Rayleigh scheme for estimating the natural frequency of conservative systems, an approximate displacement field can be assumed. In this example, the approximate bending field is assumed to be harmonic in time and sinusoidal in space:

$$w = \sum_n a_n\, Sin\omega_n t\;\; Sin\frac{n\pi y}{L} \qquad (10)$$

where the y axis is oriented along the length of the beam, w is the transverse displacement in the z direction, $\omega_n$ and $a_n$ are the natural frequency and amplitude, respectively, of the $n^{th}$ mode, L is the length of the beam, and t is time. Only the fundamental mode (n=1) is of interest in this study. Thus the only non-zero term in the bending strain field $\vec{\epsilon}^o$ is $\epsilon_2^o$, and is given as:

$$\epsilon_2^0 = z\, \frac{\pi^2}{L^2} a_1\, Sin\omega_1 t\;\; Sin\frac{\pi y}{L} \qquad (11)$$

where, z is the distance of the micro-device from the neutral axis of the beam. The only non-zero component of the actuation voltage vector is now $E_2$ and is assumed to be proportional to the output of the sensory devices, and hence, to the bending strains. Thus $E_2$ is written as:

$$E_2 = E_2^* \, Sin\omega_1 t \;\; Sin\frac{\pi y}{L} \tag{12}$$

where the amplitude $E_2^*$ is assumed to be proportional to the amplitude of the bending strain due to the fundamental vibrational mode of the beam, and the non-zero terms of the actuation strain vector are now written as:

$$\epsilon_i^T = d_{2i} \, E_2 \quad , \quad i = 1\text{-}6 \tag{13}$$

Equations (10-13) are used in Equation (8) to compute the natural frequency of the system.

## 4. RESULTS AND DISCUSSIONS

Figure (2) shows the increase in the natural frequency as a function of the excitation strain for different numbers of actuators. For convenience, the natural frequency is normalized with respect to that for zero electrical excitation. The actuation strain amplitude is normalized with respect to the far field strain. The increase in the natural frequency is a measure of the stiffening of the beam due to the actuation loads. Figure (3) illustrates the dependence of this stiffening effect, and hence the natural frequency, on the Young's modules of the host material. Figure (4) illustrates the relative contributions of the mechanical interaction energy, and the electrical energy, towards stiffening of the structure. The mechanical term is substantially smaller than the electrical term but its relative contribution increases as the actuation load or the host stiffness is increased.

The strain concentration factors in the host as a result of the far-field bending and the actuation loads can also be obtained as a result of this analysis, as shown elsewhere in the literature [Alghamdi and Dasgupta, 1993].

## 5. CONCLUSIONS

This paper has summarized recent research on the use of Eshelby's techniques for modeling the interaction between micro-devices and the surrounding host. This method presented a unified approach for addressing the interaction mechanics of micro-devices embedded in an adaptive structure. This technique also gives simultaneous information of the stress and strain concentrations in the host due to the presence of the micro-device. Such information is important for designing reliable smart structures.

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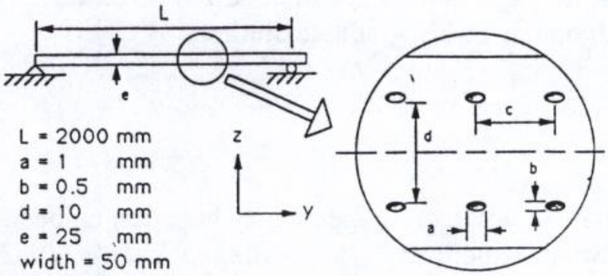


Figure 1. Adaptive Beam with Embedded Rows of Micro-devices.

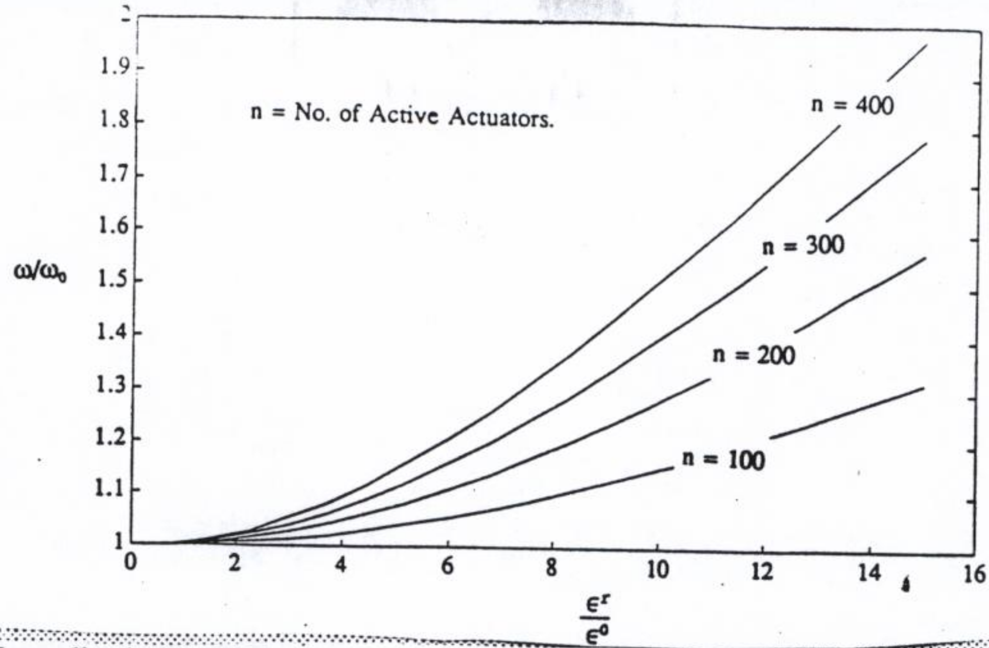


**Figure 2. Normalized Natural Frequency as a function of Normalized Actuation Strain ($\varepsilon^r_2/\varepsilon^0_2$) for different device densities.**

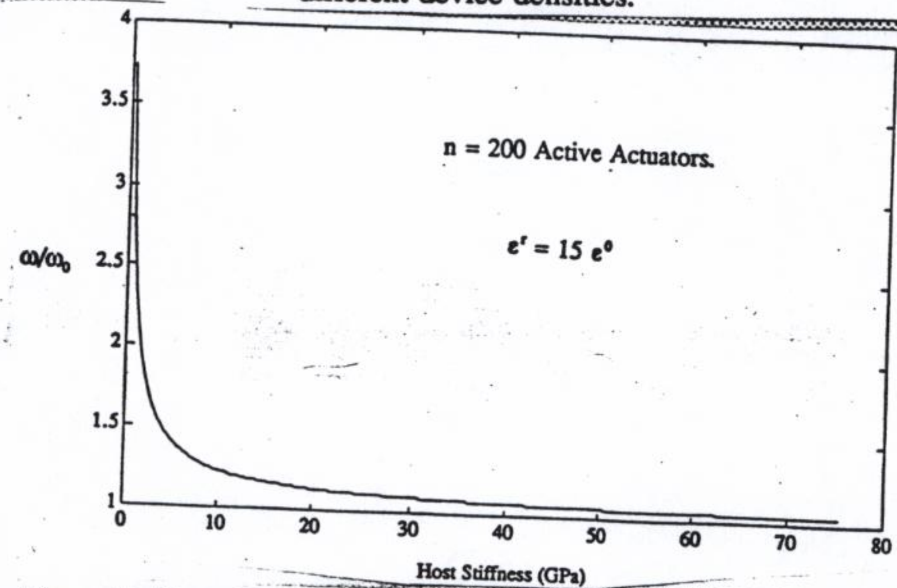


**Figure 3. Plot of Normalized Natural Frequency vs. Host Stiffness.**

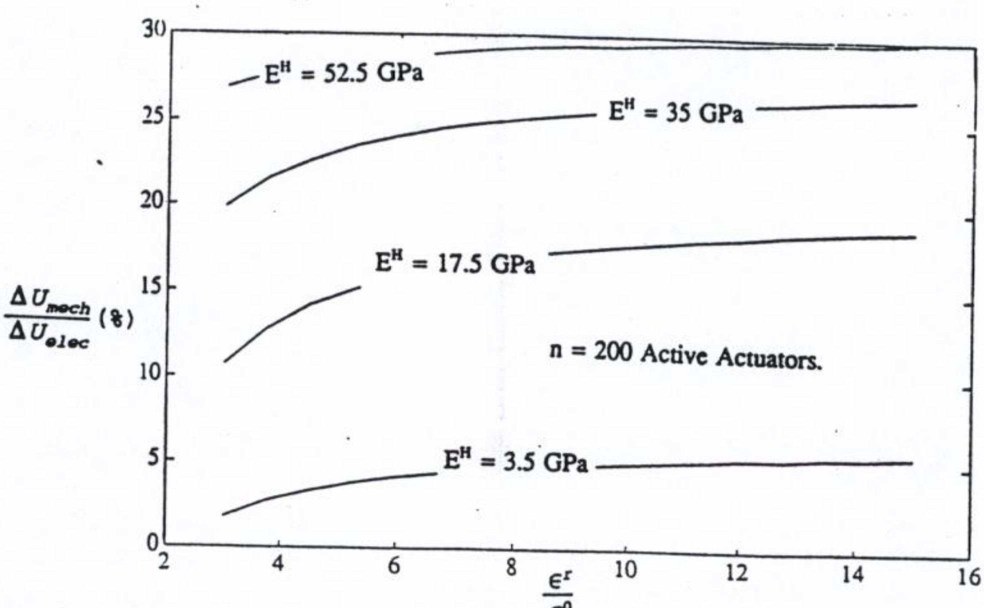


Figure 4. The percentage change in Mechanical Energy term relative to Electrical Energy term as a function of Normalized Actuation strain for different host stiffness.